String Theory 250
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Problem set # 6

1 Moduli space of hyperelliptic curve

A "hyperelliptic curve" is a (special) Riemann surface of genus $g \geq 1$ that is given by the the set of all complex numbers $z \in \mathbf{C}, w \in \mathbf{C}$ that satisfy the algebraic relation

$$w^2 = \prod_{i=1}^{2g+2} (z - e_i),$$

where $e_1, \ldots, e_{2g+2}$ are fixed, distinct complex numbers, and we also add two points at infinity (formally $z = \infty$ with $w = \pm z^{g+1}$). We can think of this Riemann surface as the complex z plane together with $z = \infty$, but with each point doubled (because w has two possible values for a given z), except for $z = e_1, \ldots, e_{2g+2}$, for which the points are not doubled. We are now going to calculate the number μ of moduli from this representation of a Riemann surface. Take z to be a **local** coordinate on the Riemann surface away from the "bad points" $e_1, \ldots, e_{2g+2}, \infty$.

- Near $z = e_i$, show that w is a good local coordinate, and near $z = \infty$, show that $1/z$ is a good local coordinate.
- Can you see, geometrically, why the genus of this Riemann surface is g?
- Now we can calculate μ by counting how many linearly independent holomorphic tensors b_{zz} there are. Assume that $b_{zz} = z^n w^m$. Find for which integers n, m this tensor b_{zz} is nonsingular. [You need to check the points $z = e_1, \ldots, e_{2g+2}$ and $z = \infty$, by going to the good local coordinates in this case.]
- Repeat the exercise for κ, the number of CKVs, by counting how many linearly independent holomorphic vectors c^z there are.

2 Anomaly of the $j = - : bc :$ current

Let (b_{ab}, c^a) be the Fadeev-Popov ghosts. Using the anomalous transformation of the $j_a = - : b_{ab} c^b :$ current, find the coefficient α in the relation $\nabla_a j^a = \alpha R$, where R is the Ricci scalar.