Higgs Boson - Lecture 3

Limits on the Higgs Mass and Radiative Corrections to m_W

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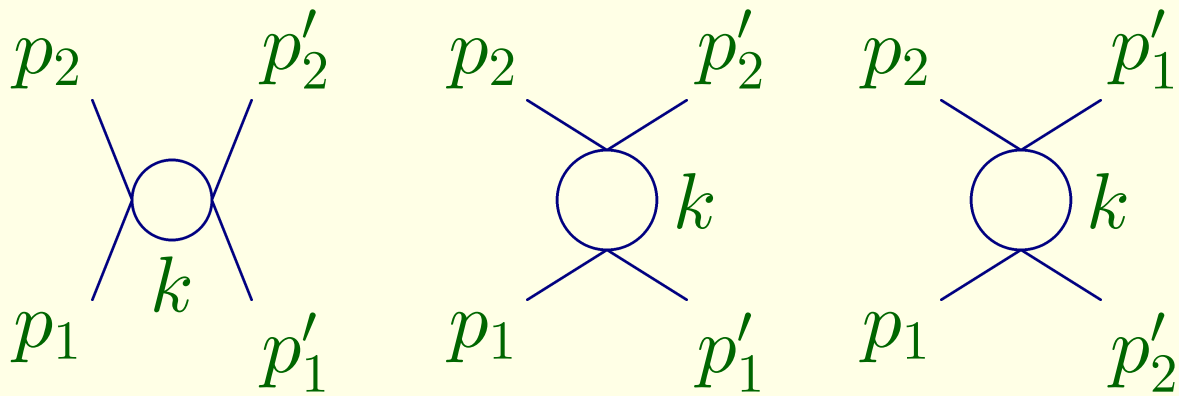
Bounds on the Higgs Mass

- A priori the Higgs mass could be anything :$m_H^2 = 2\lambda\ v^2$
- If λ is too large, perturbation theory fails
- Fix every parameter by comparison to a measurement
- If we measure α at small q^2, $\alpha \approx 1/137$.
- In muonic atoms, the muons are inside the polarization cloud
- They see "more" nuclear charge: α increases at short distance.
- In QCD we find α_s behaves oppositely.
- Let us examine what happens in scalar field theory.

Upper Bounds on the Higgs Mass



Second order scattering in ϕ^4.

Study radiative corrections to λ

Lagrangian

$$\mathcal{L} = \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{2}\lambda\phi^4$$

Two-to-two amplitude

Lowest order

$$-i\mathcal{M}_0 = -12i\lambda; \qquad [12 = 4!/2]$$

Mandelstam variables

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_1' + p_2')^2; \\ t &= (p_1 - p_1')^2 = (p_2 - p_2')^2; \\ u &= (p_1 - p_2')^2 = (p_2 - p_1')^2 \end{aligned}$$

For a real scattering event, these always statisfy

$$s + t + u = 4m^2$$

One-loop amplitude

we take $s = t = u = Q^2 < 0$ [off-shell]

$$-i\mathcal{M}_{s,2} = (-i\lambda/2)^2 \cdot (24)^2 \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-\mu^2} \frac{i}{(k-Q)^2-\mu^2}$$

forget about spontaneous break down

Use the apparent scalar mass μ.

Q is just $Q = p_1 + p_1'$ for first diagram.

Factor of $1/2$ comes from the loop

Use the Feynman trick

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{(xA+(1-x)B)^2}$$

Do integral:

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu^2}\frac{1}{(k-Q)^2-\mu^2} = \int_0^1 dx \int 1\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-2xk\cdot Q+xQ^2-\mu^2)]^2}$$

$$= \int_0^1 1dx \int 1\frac{d^4k}{(2\pi)^4}\frac{1}{[(k-xQ)^2-x^2Q^2+xQ^2-\mu^2)]}$$

Dimensional regularization:

$$\int \frac{d^np}{(p^2-m^2)^\alpha} = \frac{i\pi^{n/2}\Gamma(\alpha-n/2)}{\Gamma(\alpha)}m^{n-2\alpha}(-1)^\alpha$$

$$\int \frac{d^np\,p^2}{(p^2-m^2)^\alpha} = \frac{i\pi^{n/2}\Gamma(\alpha-n/2-1)}{\Gamma(\alpha)}m^{n-2\alpha}(-1)^{\alpha+1}m^2\frac{n}{2}$$

Remember: $\Gamma(n+1)=n!,\ \ z\Gamma(z)=\Gamma(z+1)$

Using dimensional regularization

$$-i\mathcal{M}_{s,2} = (-i\lambda/2)^2 \cdot (24)^2 \cdot \frac{-i}{16\pi^2} \int_0^1 dx \Gamma(\epsilon)(\mu^2 + x^2Q^2 - xQ^2)^{-\epsilon}$$

where

$$\epsilon = 2 - \frac{n}{2}$$

$A^\epsilon = e^{\epsilon \ln A} = 1 + \epsilon \ln A + \ldots.$

Two other diagrams:

$$\mathcal{M}_0 + \mathcal{M}_2 = 12\lambda - \frac{27\lambda^2}{2\pi^2} \int_0^1 dx [\Gamma(\epsilon) - \ln(\mu^2 + x^2Q^2 - xQ^2)]$$

Renormalize

"Oh, I meant that the amplitude should be 12λ, when $Q^2 = -\mu^2$:

add a piece $\delta\lambda$ to fix this up."

$$\mathcal{M}_0 + \mathcal{M}_2 = 12\lambda + 12\delta\lambda - \frac{27\lambda^2}{2\pi^2}\int_0^1 dx[\Gamma(\epsilon) - \ln(\mu^2 + x^2Q^2 - xQ^2)]$$

where

$$12\delta\lambda - \frac{27\lambda^2}{2\pi^2}\int_0^1 dx[\Gamma(\epsilon) - \ln(\mu^2 - x^2\mu^2 + x\mu^2)] = 0$$

c'est a dire

$$\begin{aligned}\mathcal{M}_0 + \mathcal{M}_2 &= 12\lambda + \frac{27\lambda^2}{\pi^2}\int_0^1 dx \ln\frac{\mu^2 + x^2Q^2 - xQ^2}{\mu^2 - x^2\mu^2 + x\mu^2} \\ &\approx 12\lambda + \frac{27\lambda^2}{2\pi^2}\ln\frac{Q^2}{\mu^2}\end{aligned}$$

Sum geometric series

$$\begin{aligned}
\mathcal{M} &= 12\lambda[1 + \frac{9\lambda}{8\pi^2}\ln\frac{Q^2}{\mu^2} + \ldots \\
&= \frac{12\lambda}{1 - \frac{9\lambda}{4\pi^2}\ln\frac{Q}{\mu}} \\
&\equiv 12\lambda_Q
\end{aligned}$$

Landau pole: blows up at

$$Q = \Lambda_{\mathrm{Landau}} = \mu e^{4\pi^2/(9\lambda)}$$

Interpretation: theory doesn't exist beyond here!

Implies upper bound on Higgs mass

Assume

$$m_H < \frac{1}{2}\Lambda_{\rm Landau}; \ln(\Lambda_{Landau}/m_H) = \frac{4\pi^2}{9\lambda}$$

from

$$m_H^2 = 2\lambda v^2 < \frac{2\times 4\pi^2 v^2}{9\ln 2}$$

which gives

$$m_H < 875 \text{ GeV}$$

Better: four scalars

$$m_H^2 = 2\lambda v^2 < \frac{4\pi^2 v^2}{3 \ln 2}$$

which gives

$$m_H < 1.07 \text{ TeV}.$$

Alternatively: suppose theory works up to new scale Λ

$$m_H^2 = 2\lambda v^2 < \frac{4\pi^2 v^2}{3 \ln \Lambda/v}$$

$$\Lambda = 10^{16} \text{ GeV}, \quad m_H = 160 \quad \text{GeV};$$
$$\Lambda = 10^{10} \text{ GeV}, \quad m_H = 213 \quad \text{GeV}$$

Lower Bounds on the Higgs Mass

Generalize our treatment with differential equation

$$\frac{1}{\lambda(Q)} \equiv \frac{1}{\lambda_Q} = \frac{1}{\lambda} - \frac{9}{4\pi^2} \ln \frac{Q}{\mu}$$

so that

$$\frac{1}{\lambda(Q)^2} \frac{\partial \lambda(Q)}{\partial \ln Q} = \frac{9}{4\pi^2}$$

Add effect of t quark

$$\begin{aligned} \lambda &= \frac{m_H^2}{2v^2} \\ g_t &= -\frac{m_t}{v} \end{aligned}$$

the equation for λ reads, with $t = \ln Q^2$,

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left[12\lambda^2 + 12\lambda g_t^2 - 12 g_t^4 - \frac{3}{2}\lambda(3g^2 + g'^2) + \frac{3}{16}(2g^4 + (g^2 + g'^2)^2)\right]$$

If λ is small, t quark can drive it negative

$\lambda < 0$ is unstable

$$\lambda(\Lambda) = \lambda(v) - \tfrac{3}{4\pi^2} g_t^4 \ln(\Lambda^2/v^2)$$

so this sets a limit

$$m_H^2 > \tfrac{3v^2}{2\pi^2} g_t^4 \ln(\Lambda^2/v^2) \approx (68\ \mathrm{GeV}\sqrt{\ln(\Lambda/v)})^2$$

More detailed analysis considers the likelihood that at high temperature universe - or pieces of it - will tunnel to wrong vacuum.

Radiative Corrections

Special relation:

$$m_W = \cos\theta_W m_Z$$

Consequence of Higgs doublets only:

$$(D_\mu\langle\phi\rangle)^\dagger D_\mu\langle\phi\rangle$$

If we have several multiplets we can write

$$D_\mu\langle\phi\rangle \to \sum_\phi \left[\frac{g}{\sqrt{2}}(T^+W_\mu^+ + T^-W_\mu^-) + gT_3W_3 - g'T_3B\right]\langle\phi\rangle$$

Used $\langle\phi\rangle = (T_3 + Y/2)\langle\phi\rangle = 0$

Now we can rewrite masses as

$$(D_\mu\langle\phi\rangle)^\dagger D_\mu\langle\phi\rangle =$$
$$\sum_\phi \left\{\frac{g^2}{2}[T(T+1) - T_3^2]2W^+W^- + T_3^2(g^2 + g'^2)Z^2\right\}\langle\phi\rangle^2$$

The ratio of the squares of the masses is thus

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2+g'^2} \frac{\sum_\phi [T(T+1)-T_3^2]\frac{1}{2}\langle\phi\rangle^2}{T_3^2\langle\phi\rangle^2}$$

If all Higgs multiplets have weak isospin 1/2, relation preserved

Fixing the parameters of Standard Model

- Three fundamental parameters of electroweak theory: g, g', v
- Determine these with three physical measurements
  - $m_Z = 91.1876(21)\ \ \mathrm{GeV}$
  - $G_F = 1.16639(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$
  - $\alpha_{em} = 1/137.03599975(50)$; $\alpha(m_Z)$ not so well known
- Other "external" parameters: m_t, m_H, CKM matrix,...
- Every observable can be predicted in terms of these parameters
- Prime example: $m_W = 80.422(47)\ \ \mathrm{GeV}$

Basic relation for W mass

$$
\begin{aligned}
m_Z^2 &= \frac{(g^2+g'^2)v^2}{4} \\
G_F &= \frac{1}{\sqrt{2}v^2} \\
\frac{1}{e^2} &= 1/(4\pi\alpha) = \frac{1}{g^2} + \frac{1}{g'^2}
\end{aligned}
$$

Cannot also demand first order relation $m_W^2 = \frac{g^2v^2}{4}$

Instead

$$
m_W^2 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}G_F m_Z^2}}\right) m_Z^2
$$

Role of Δr

All radiative corrections reside in Δr

Sensitivity of m_W^2 to Δr:

$$
\begin{aligned}
m_W^2 &= \frac{1}{2}\left(1+\sqrt{1-\sin^2 2\theta_W(1+\Delta r)}\right) m_Z^2 \\
\frac{1}{m_W^2}\frac{\partial m_W^2}{\partial \Delta r} &= -\frac{\sin^2\theta_W}{1-2\sin^2\theta_W} \approx -0.36 \\
\frac{1}{m_W}\frac{\partial m_W}{\partial \Delta r} &\approx -0.18
\end{aligned}
$$

shift of Δr by 0.01 changes m_W by 145 MeV.

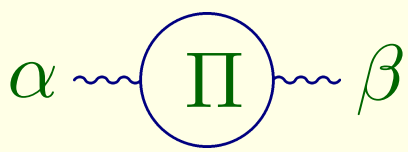
Δr gets contributions from loops of fermions, Higgs, Ws Zs

Role of Δr

$$\Delta r = \Delta\alpha - \Delta\rho\frac{c^2}{s^2} + \frac{g^2}{s^2}[\Pi'_{33}(m_Z^2) - 2s^2\Pi'_{3Q}(m_Z^2)] + \frac{g^2}{s^2}(s^2 - c^2)\Pi'_{11}(m_W^2)$$

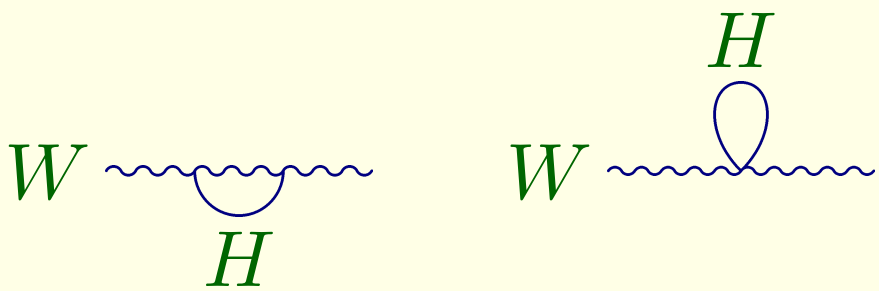
where $s = \sin\theta_W$, $c = \cos\theta_W$, and where

$$\begin{aligned}
\Delta\alpha &= e^2[\Pi_{QQ}(m_Z^2) - \Pi_{QQ}(0)] \\
\Delta\rho &= \frac{g^2}{m_W^2}[\Pi_{11}(0) - \Pi_{33}(0)] \\
\Pi'(q^2) &= [\Pi(q^2) - \Pi(0)]/q^2
\end{aligned}$$



Π_{33} represents loop with W_3 in and W_3 out, etc.

Higgs Contribution to Δr



Higgs contributions to W and Z self-energies.

- Seek only leading dependence on m_H
- No coupling to photon so drop terms with Q coupling
- Approximate $\Pi'_{33}(m_Z^2) = \Pi'_{11}(m_W^2) = \Pi'_{33}(0)$

Result:

$$\Delta r_H = \frac{c^2}{s^2}\frac{g^2}{m_W^2}[\Pi_{33}(0) - \Pi_{11}(0)] + 2g^2\Pi'_{33}(0)$$

Diagram with emission and absorption at same point doesn't contribute:

$$\Pi_{33}(0) = \Pi_{11}(0); \qquad \Pi'_{33}(0) = 0$$

Couplings of Higgs to WW and ZZ:

$$\begin{aligned} WWH : (ig^2 v/2) g_{\alpha\beta} &= igm_W g_{\alpha\beta} \\ ZZH : (ig^2 v/2\cos^2\theta_W) g_{\alpha\beta} &= (igm_W/\cos^2\theta_W) g_{\alpha\beta} \end{aligned}$$

α and β tied to the gauge bosons

$1/\cos\theta^2$ goes away because each W_3 has only $\cos\theta$ worth of Z in it.

For either Π_{11} or Π_{33} we have

$$\begin{aligned} i\Pi_{\alpha\beta}(q^2) &= (im)^2 \int \frac{d^n k}{(2\pi)^4} \frac{i}{(k+q)^2 - m_H^2} \frac{-i(g_{\alpha\beta} - k_\alpha k_\beta/m^2)}{k^2 - m^2} \\ &= (im)^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^4} \frac{g_{\alpha\beta} - (k' - xq)_\alpha (k' - xq)_\beta/m^2}{(k'^2 - x^2 q^2 + xq^2 - xm_H^2 - (1-x)m^2)^2} \end{aligned}$$

Look only at $g_{\alpha\beta}$ type terms:

$$k'_\alpha k'_\beta \to k'^2 g_{\alpha\beta}/n$$

to obtain

$$i\Pi(q^2) = -\int_0^1 dx \int \frac{d^n k'}{(2\pi)^4} \frac{m^2 - k'^2/n}{(k'^2 - \mu^2)^2}$$

where m is m_Z or m_W and

$$\mu^2 = x m_H^2 + (1-x)m^2 - x(1-x)q^2$$

Dimensional regularization again

$$\begin{aligned} i\Pi(q^2) &= \frac{-i\Gamma(2-n/2)}{16\pi^2} \int_0^1 dx \frac{m^2 + \mu^2/(2-n)}{(\mu^2)^{2-n/2}} \\ i\frac{\partial \Pi(q^2)}{\partial q^2} &= \frac{-i\Gamma(2-n/2)}{16\pi^2} \int_0^1 dx \left[\frac{(2-n/2)m^2/\mu^2 - 1/2}{(\mu^2)^{2-n/2}} \right] [-x(1-x)] \end{aligned}$$

We expand

$$(\mu^2)^{2-n/2} = 1 + (2-n/2)\ln\mu^2 = 1 + (2-n/2)[\ln m_H^2 + \mathcal{O}(m^2/m_H^2)$$

so

$$\begin{aligned} i\Pi(q^2) &= \frac{-i\Gamma(2-n/2)}{16\pi^2}\int_0^1 dx[m^2+\mu^2/(2-n)][1-(2-n/2)\ln m_H^2] \\ i\Pi(0) &= \frac{i\Gamma(2-n/2)}{16\pi^2}[m^2+\frac{m_H^2+m^2}{4-2n}][1-(2-n/2)\ln m_H^2] \end{aligned}$$

and

$$\begin{aligned} i(\Pi_{33}(0)-\Pi_{11}(0)) &= \frac{-i\Gamma(2-n/2)}{16\pi^2}(m_Z^2-m_W^2) \\ &\quad \times\frac{1}{16\pi^2}[1+\frac{1}{4-2n}][1-(2-n/2)\ln m_H^2] \end{aligned}$$

Look only at m_H dependence.

Total result isn't even finite since we neglected W and Z loops

$$\Pi_{33}(0) - \Pi_{11}(0) = \tfrac{3}{4}(m_Z^2 - m_W^2)\tfrac{1}{16\pi^2}\ln m_H^2$$

In Π' we drop the term suppressed by m^2/m_H^2

$$\begin{aligned} i\frac{\partial\Pi(q^2)}{\partial q^2} &= \frac{-i\Gamma(2-n/2)}{16\pi^2}\int_0^1 dx[x(1-x)/2][1-(2-n/2)\ln m_H^2] \\ &\to \frac{i}{16\pi^2}\frac{1}{12}\ln m_H^2 \end{aligned}$$

Combining all this,

$$\Delta r_H = \frac{c^2}{s^2}\frac{g^2}{m_W^2}[\Pi_{33}(0) - \Pi_{11}(0)] + 2g^2\Pi'_{33}(0)$$

$$
\begin{aligned}
&= \frac{g^2}{16\pi^2}[\frac{3}{4}+\frac{2}{12}]\ln m_H^2 \\
&= \frac{\sqrt{2}G_F m_W^2}{4\pi^2}\frac{11}{12}\ln m_H^2 \\
&\approx 2.5\times 10^{-3}\ln m_H^2
\end{aligned}
$$

t Contribution to Δr

Contribution from degenerate quark doublets small

- $\Delta\rho$ vanishes

- Ignore difference between m_Z and m_W

$$\frac{g^2}{s^2}[\Pi'_{33}(m_Z^2) - 2s^2\Pi'_{3Q}(m_Z^2)] + \frac{g^2}{s^2}(s^2 - c^2)\Pi'_{11}(m_W^2) = -2s^2\Pi'_{3Y/2}(m_Z^2)$$

vanishes when we sum over two quarks, opposite T_3, same Y

Big contribution from t

$$\Delta r_t = -\frac{3G_F c^2}{8\pi^2\sqrt{2}s^2}\left\{m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\frac{m_t^2}{m_b^2}\right\} = -0.036\left(\frac{m_t}{175\ \mathrm{GeV}}\right)^2$$

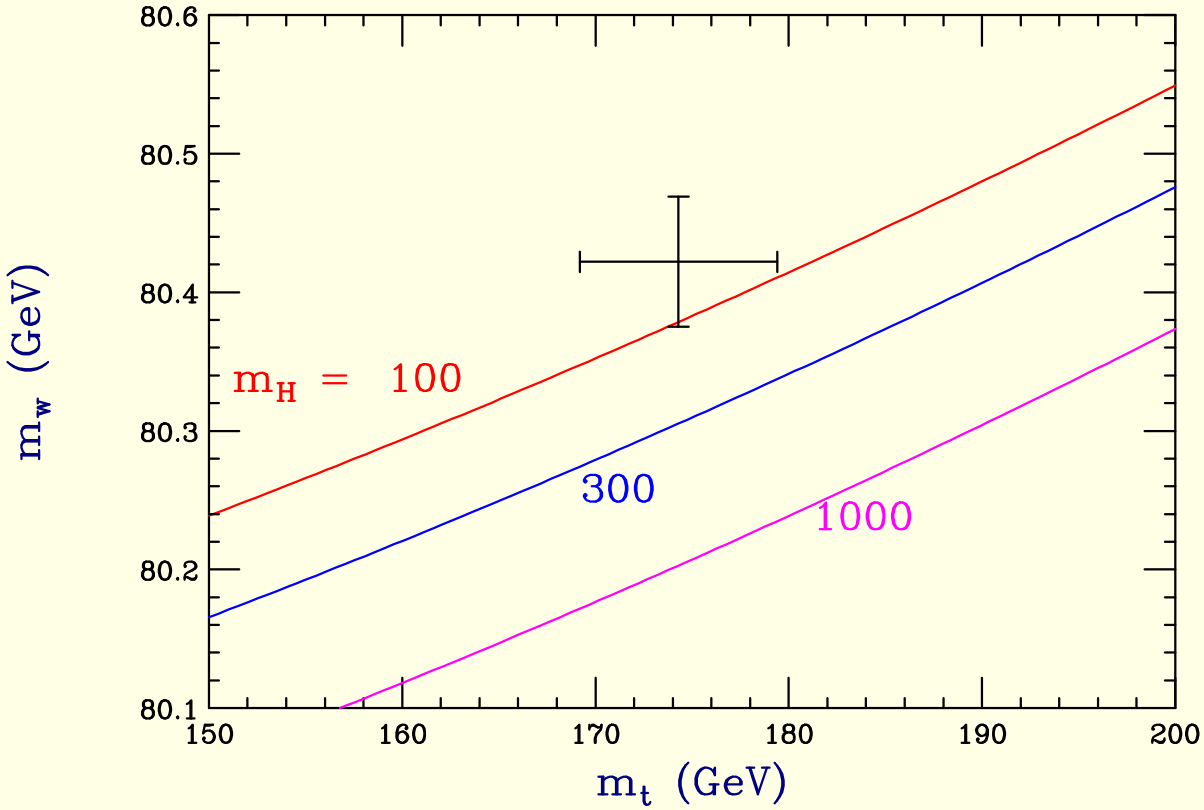
Combined effect of t and Higgs

$$\begin{aligned}\frac{1}{m_W}\delta m_W &= -0.18\delta\Delta r = -0.18[5.0\times 10^{-3}\delta\ln m_H - 0.072\frac{\delta m_t}{m_t}] \\ &= -9.0\times 10^{-4}\delta\ln m_H + 1.3\times 10^{-2}\frac{\delta m_t}{m_t}\end{aligned}$$

This is first order. Fuller treatment gives interpolating formula

$$\begin{aligned}m_W(\mathrm{GeV}) &= 80.3829 - 0.0579\ln(m_H/100) - 0.008[\ln(m_H/100)]^2 \\ &\quad -0.517\left(\frac{\delta\alpha_H^{(5)}}{0.0280}-1\right) + 0.543\left[\left(\frac{m_t}{175}\right)^2 - 1\right] \\ &\quad -0.085\left(\frac{\alpha_s(M_Z)}{0.118}-1\right)\end{aligned}$$

Current Indications for m_H

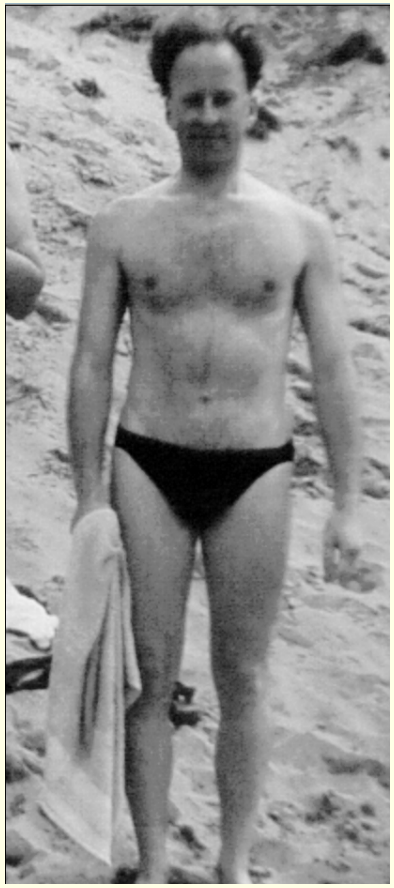


Current values for m_t and m_W, with their uncertainties, with curves determined by radiative corrections

Summary

- We know everything about the Higgs boson but
  - Its mass
  - Whether it exists
- Simple Higgs is model for alternatives
  - Maybe multiple Higgs - Supersymmetry
  - May not have particle character - heavy higgs
  - May be dynamical
- Cette chasse reste la chose la plus importante pour la physique des hautes energies.

Flash! Bare Higgs Revealed.



P.H. at the Firth of Forth, July, 1960.
Photograph by J. D. Jackson