Higgs Boson - Lecture 2

Higgs Decay and Production

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Le Higgs: La Chasse Continue!

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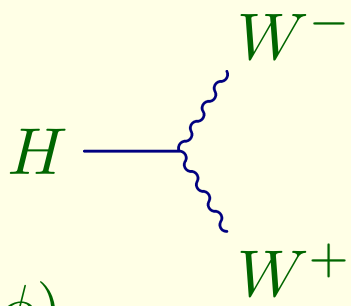
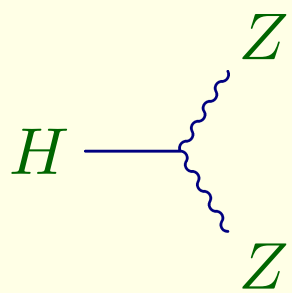
Higgs Decays

- Partial widths dictated by Lagrangian
- Observability important, not just branching ratio
- Even one-loop decays important
- General rule: Higgs prefer heft

$H \to WW, ZZ$





From $(D_\mu\phi)^\dagger(D^\mu\phi)$

$$\frac{1}{2}\left[\frac{g^2}{2}W_\mu^+W^{-\mu}(v+\rho)^2 + \partial_\mu\rho\partial^\mu\rho + \frac{g^2}{4\cos^2\theta_W}Z_\mu Z^\mu(v+\rho)^2\right]$$

$H \to W^+W^-$ has the matrix element

$$-i\mathcal{M} = i(g^2v/2)\epsilon_+^*\epsilon_-^* = igm_W\epsilon_+^*\epsilon_-^*$$

Polarization sum:

$$\sum_{\epsilon_+} \epsilon^*_{+\mu}\epsilon_{+\nu} = -g_{\mu\nu} + \frac{p^+_\mu p^+_\nu}{m_W^2}$$

Matrix element squared:

$$\begin{aligned} |\mathcal{M}|^2 &= g^2 m_W^2 \left(g_{\mu\nu} - \frac{p^+_\mu p^+_\nu}{m_W^2}\right)\left(g^{\mu\nu} - \frac{p^{-\mu}p^{-\nu}}{m_W^2}\right) \\ &= \frac{g^2 m_H^4}{4m_W^2}\left(1 - 4\frac{m_W^2}{m_H^2} + 12\frac{m_W^4}{m_H^4}\right) \end{aligned}$$

Partial width:

$$\begin{aligned} \Gamma(H \to WW) &= \frac{1}{8\pi}\frac{p_{cm}}{M^2}|\mathcal{M}|^2 = \frac{G_F m_H^3}{8\pi\sqrt{2}}\sqrt{1 - \frac{4m_W^2}{m_H^2}}\left(1 - 4\frac{m_W^2}{m_H^2} + 12\frac{m_W^4}{m_H^4}\right) \\ \Gamma(H \to ZZ) &= \frac{G_F m_H^3}{16\pi\sqrt{2}}\sqrt{1 - \frac{4m_Z^2}{m_H^2}}\left(1 - 4\frac{m_Z^2}{m_H^2} + 12\frac{m_Z^4}{m_H^4}\right) \end{aligned}$$

$H \to WW^*, ZZ^*$

Below WW threshold Higgs can still go to W^*W^*, Z^*Z^*

$$\Gamma(H \to ZZ^*) = \frac{G_F m_H m_Z^2}{8\pi^2\sqrt{2}} \frac{\Gamma_Z}{m_z} F(\frac{m_Z^2}{m_H^2})$$

where

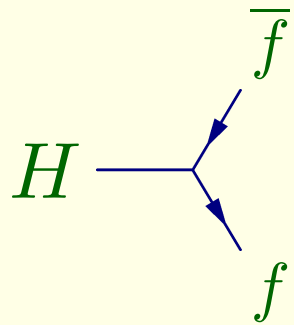
$$\begin{aligned} F(u) &= \frac{3(1-8u+20u^2)}{\sqrt{4u-1}} \cos^{-1}\left(\frac{3u-1}{2u^{3/2}}\right) \\ &\quad -(1-u)(\left(\frac{47}{2}u - \frac{13}{2} + \frac{1}{u}\right) \\ &\quad -\frac{3}{2}(1-6u+4u^2)\ln x \end{aligned}$$

OK if not too close to threshold

OK if not too far from threshold

Similar for $H \to WW^*$

Higgs decays to fermion pairs



The coupling of the Higgs boson to fermion antifermion pairs is

$$\frac{g_f}{\sqrt{2}} H \overline{f} f$$

while

$$m_f = -\frac{g_f v}{\sqrt{2}} = -\frac{2 g_f m_w}{\sqrt{2} g}$$

so the coupling is equivalently

$$-\frac{g m_f}{2 m_W} H \overline{f} f$$

So

$$-i\mathcal{M} = -i \frac{g m_f}{2 m_w} \overline{u}(p') v(p)$$

The square, summed over final state spins is

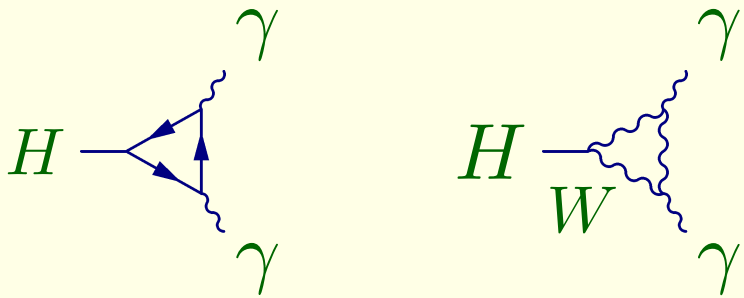
$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{g^2 m_f^2}{4m_W^2}\mathrm{Tr}\ (\not{p}' + m_f)(\not{p} - m_f) \\
&= \frac{g^2 m_f^2}{4m_W^2} 4(p' \cdot p - m_f^2) =_f \frac{g^2 m_f^2}{2m_W^2}(m_H^2 - 4m_f^2).
\end{aligned}
$$

$$
\Gamma(H \to f\overline{f}) = \frac{G_F}{\sqrt{2}} \frac{m_f^2 m_H}{4\pi} \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2} C_f
$$

where the color factor C_f is 1 for leptons and 3 for quarks.

$$
\begin{aligned}
\Gamma(H \to b\bar{b}) &= 4.0\ \mathrm{MeV} \left(\frac{m_H}{100\ \mathrm{GeV}}\right) \left(1 - \frac{4m_b^2}{m_H^2}\right)^{3/2} \\
\Gamma(H \to t\bar{t}) &= 24\ \mathrm{GeV} \left(\frac{m_H}{400\ \mathrm{GeV}}\right) \left(1 - \frac{4m_t^2}{m_H^2}\right)^{3/2}
\end{aligned}
$$

One loop: $H \to gg, \gamma\gamma$



No direct coupling $H \to \gamma\gamma$. Proper form is

$$\mathcal{L} \propto H F_{\mu\nu} F^{\mu\nu}$$

Dimension 5 $[H \propto m, F \propto m^2]$

Not in Lagrangian, must be finite

Structure of the amplitude

$$\mathcal{M} = A(\epsilon_{1\mu}k_{1\nu} - \epsilon_{1\nu}k_{1\mu})(\epsilon^{2\mu}k^{2\nu} - \epsilon^{2\nu}k^{2\mu})$$

Do polarization sum

$$\begin{aligned} \mathcal{M} &= -2A\vec{\epsilon}_1 \cdot \vec{\epsilon}_2 k_1 \cdot k_2 \\ \sum_{pol} |\mathcal{M}|^2 &= 2|A|^2 m_H^4 \end{aligned}$$

Divide by two for identical particles

$$\Gamma = \frac{1}{16\pi}|A|^2 m_H^3$$

W and fermion loops both written in terms of complex function

$$I(z) = 3\int_0^1 dy \int_0^{1-y} dt \frac{1-4yt}{1-yt/z}$$

Note $I(z) \to 1$ as $z \to \infty$

$$
\begin{aligned}
I(z) &= 3[2z + (1-4z)f(z)] \\
f(z) &= 2z\left(\sin^{-1}\frac{1}{2\sqrt{z}}\right)^2; \quad z > 1/4 \\
&= -2z\left(-\cosh^{-1}\frac{1}{2\sqrt{z}} + \frac{i\pi}{2}\right); \quad z < 1/4
\end{aligned}
$$

W loop with $\lambda = m_W^2/m_H^2$

$$A_W = \frac{ge^2}{16\pi^2 m_W}\left[\frac{2(2\lambda-1)I(\lambda)+10\lambda-1}{4\lambda-1}\right]$$

The contribution from each fermion loop is

$$A_F = -\frac{2}{3}\frac{ge^2}{16\pi^2 m_W} Q_f^2 C_f I(\frac{m_f^2}{m_H^2})$$

Q_f is the charge of fermion f;

C_f one for charged leptons and 3 for quarks.

Full result

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 G_F m_H^3}{128\pi^3\sqrt{2}} \left| \text{“}7'' - \frac{4}{3}\sum_f Q_f^2 C_f I(\frac{m_f^2}{m_H^2}) \right|^2$$

where

$$\text{“}7'' = \frac{4(2\lambda-1)I(\lambda)+20\lambda-2}{4\lambda-1}$$

In the limit of $m_H << m_W$, “$7''$ $\to 7$.

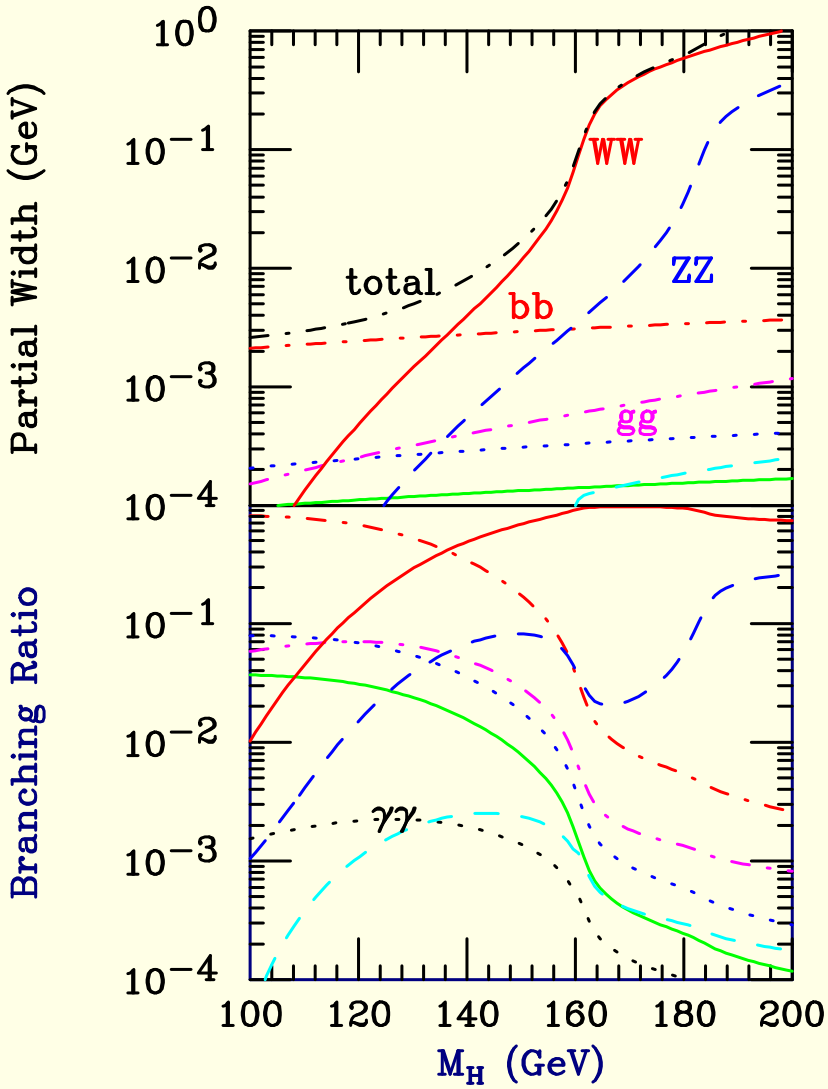
$H \to gg$

$$
\begin{aligned}
e^2 &\to g_s^2 Tr \frac{\lambda^i}{2}\frac{\lambda^j}{2} \\
e^4 &\to (\frac{1}{2}g_s^2\delta^{ij})(\frac{1}{2}g_s^2\delta^{ij}) = 2g_s^4
\end{aligned}
$$

so

$$
\begin{aligned}
\Gamma(H \to gg) &= \frac{\alpha_s^2 G_F m_H^3}{36\pi^3\sqrt{2}} |\sum_q I\left(\frac{m_q^2}{m_H^2}\right)|^2 \\
&= 74 \text{ keV } \left(\frac{\alpha_s}{0.1}\right)^2 \left(\frac{m_H}{100 \text{ GeV}}\right)^3 |\sum_q I\left(\frac{m_q^2}{m_H^2}\right)|^2
\end{aligned}
$$

Enhanced by QCD radiative corrections



Higgs Width and Branching Ratios

- WW and ZZ dominate above threshold
- WW^* dominates above 135 GeV
- BR($\gamma\gamma$) $\mathcal{O}(10^{-3})$ below WW threshold

Higgs Boson Production

Resonant Production at Lepton Colliders

Resonant production $ab \to R$ - Breit-Wigner formula:

$$\begin{aligned}\sigma &= \frac{2J+1}{(2S_a+1)(2S_b+1)}\frac{4\pi}{k^2}\frac{\Gamma^2/4}{(E-M)^2+\Gamma^2/4}BR(R\to ab)\\ &= \frac{2J+1}{(2S_a+1)(2S_b+1)}\frac{4\pi}{k^2}\frac{s\Gamma^2}{(s-m^2)^2+m^2\Gamma^2}BR(R\to ab)\end{aligned}$$

k is the incident c.m. momentum.

$$\Gamma(H\to e^+e^-) = 1.7\times 10^{-11}\left(\frac{m_H}{100\mathrm{GeV}}\right)\ \mathrm{GeV}$$

The peak the cross section is

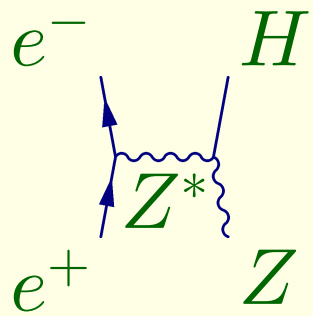
$$\sigma(e^+e^- \to H) \quad = \quad 8.3 \text{ fb} \left(\frac{100 \text{ GeV}}{m_H}\right) \times \frac{1\text{MeV}}{\Gamma_H}$$

Try muons instead:

$$\sigma(\mu^+\mu^- \to Higgs) = 360 \text{ pb} \left(\frac{(100\text{GeV})}{m_H}\right) \left(\frac{1 \text{ MeV}}{\Gamma_H}\right)$$

Doable, but late..

Associated Production of H with Z in e^+e^-

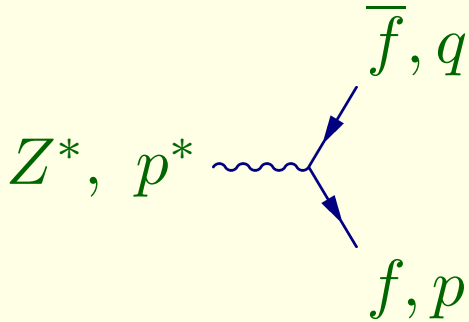


The LEP-II search focused on $e^+e^- \to ZH$

$$\begin{aligned}\sigma(e^+e^- \to ZH) &= \frac{3}{4}\frac{16\pi}{s}\frac{s\Gamma^2}{(s-m_Z^2)^2}\frac{\Gamma(Z^* \to e^+e^-)}{\Gamma}\frac{\Gamma(Z^* \to ZH)}{\Gamma} \\ &= 12\pi\frac{\Gamma(Z^* \to e^+e^-)\Gamma(Z^* \to ZH)}{(s-m_Z^2)^2}\end{aligned}$$

(Breit-Wigner formula)

$\Gamma(Z^* \to f\overline{f})$



$$
\begin{aligned}
\mathcal{M}(Z^* \to e^+e^-) &= \epsilon_\mu \overline{u}\gamma^\mu(g_V + g_A\gamma_5)v \\
|\mathcal{M}|^2_{ave} &= \frac{1}{3}[-g_{\mu\nu}]\left(\mathrm{Tr}\ \not{p}\gamma^\mu(g_V + g_A\gamma_5)\not{q}\gamma^\nu(g_V + g_A\gamma_5)\right) \\
&= -\frac{1}{3}\mathrm{Tr}\ \not{p}\gamma^\mu\not{q}\gamma_\mu(g_V^2 + g_A^2) \\
&= \frac{8}{3}p\cdot q(g_V^2 + g_A^2) \\
&= \frac{4m_Z^{*2}}{3}(g_V^2 + g_A^2)
\end{aligned}
$$

Note that we dropped the second part of the polarization sum

$$\sum_{pol} \epsilon_\mu \epsilon_\nu^* = -(g_{\mu\nu} - k_\mu k_\nu / M^2)$$

because massless fermions give conserved V and A currents

Use magic formula for the neutral current

$$\frac{g}{\cos\theta_W}(T_3 - Q\sin^2\theta_W) \to \frac{g}{\cos\theta_W}(T_3\gamma_\mu\frac{1}{2}(1-\gamma_5) - Q\gamma_\mu\sin^2\theta_W)$$

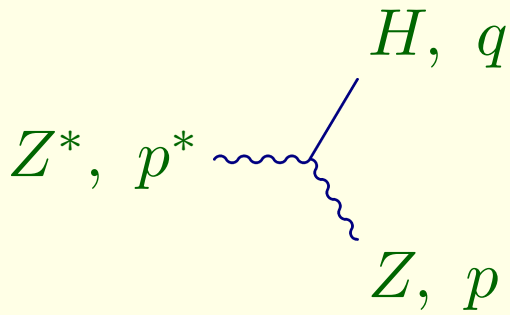
so that

$$g_V = \frac{g}{\cos\theta_W}(\frac{1}{2}T_{3L} - Q\sin^2\theta_W); \quad g_A = -\frac{g}{\cos\theta_W}\frac{1}{2}T_{3L}$$

Treating the final fermions as massless

$$\Gamma(Z \to f\overline{f}) \quad = \quad \frac{\alpha m_Z^*}{48}\frac{1}{\sin^2\theta_W\cos^2\theta_W}[(2T_{3L} - 4Q\sin^2\theta_W)^2 + 4T_{3L}^2]$$

$Z^* \to ZH$



Lagrangian contains

$$\frac{g^2 v H Z_\mu Z^\mu}{4\cos^2\theta_W} = \frac{g m_Z H Z_\mu Z^\mu}{2\cos\theta_W}$$

giving a vertex factor

$$i\frac{g m_Z g_{\mu\nu}}{\cos\theta_W}$$

$$\begin{aligned}
|\mathcal{M}|^2_{ave} &= \frac{1}{3}\left(\frac{g m_Z}{\cos\theta_W}\right)^2 \left(g_{\mu\nu} - p^*_\mu p^*_\nu / m^{*2}\right)\left(g^{\mu\nu} - p^\mu p^\nu / m^2\right) \\
&= \frac{1}{3}\left(\frac{g m_Z}{\cos\theta_W}\right)^2 [\frac{p^2_{cm}}{m^2} + 3]
\end{aligned}$$

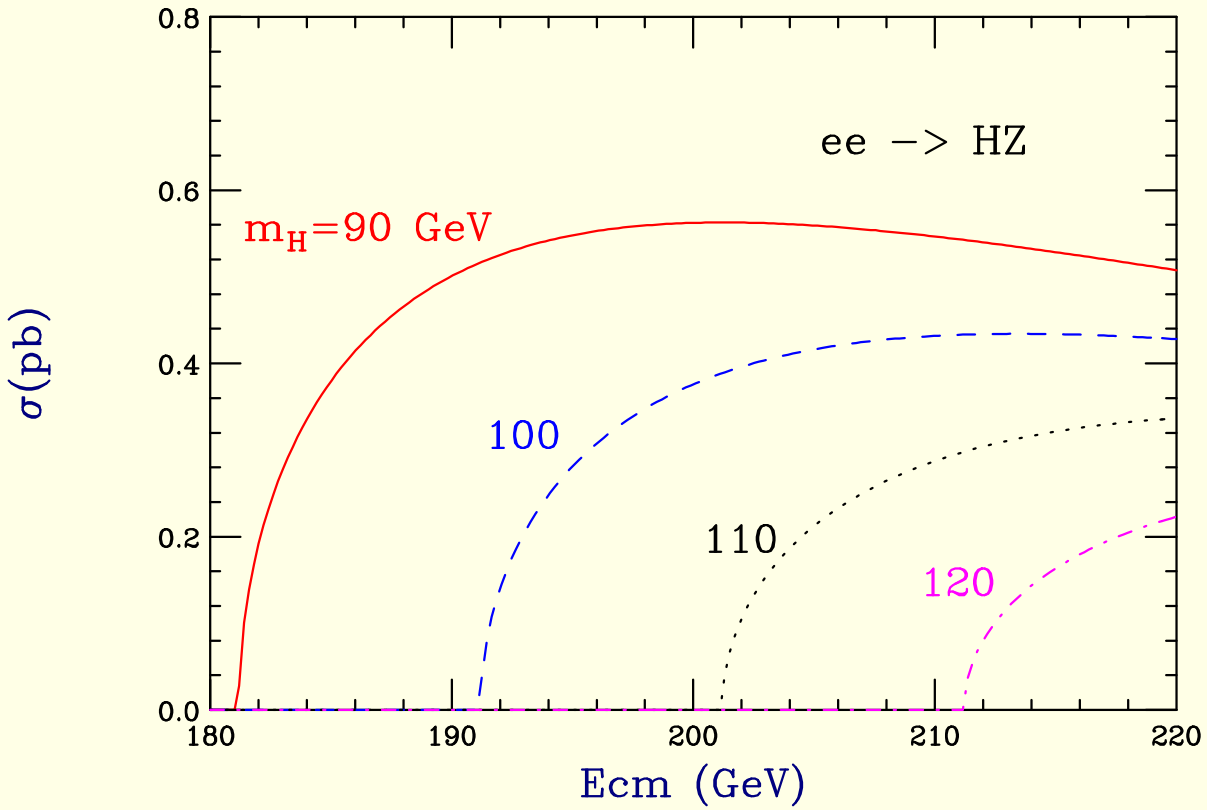
The partial width then for $Z^* \to ZH$ is

$$\begin{aligned}\Gamma(Z^* \to ZH) &= \frac{1}{24\pi}\left(\frac{g m_Z}{\cos\theta_W}\right)^2 [\frac{p_{cm}^2}{m^2} + 3]\frac{p_{cm}}{m^{*2}} \\ &= \frac{\alpha p_{cm}}{6\sin^2\theta_W\cos^2\theta_W}\left[\frac{p_{cm}^2}{m^{*2}} + 3\frac{m^2}{m^{*2}}\right]\end{aligned}$$

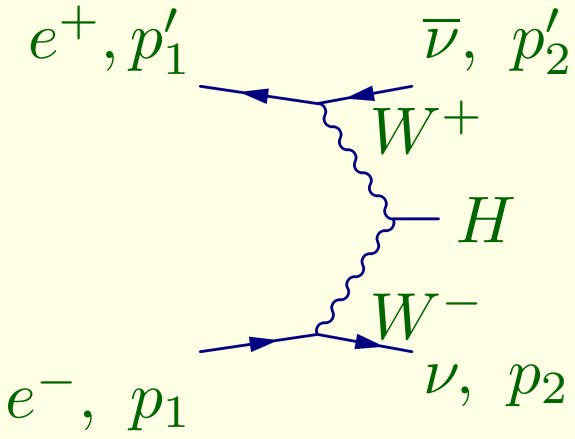
Combining the partial widths to find $\sigma(e^+e^- \to ZH)$:

$$\begin{aligned}&\frac{12\pi}{(s-m_Z^2)^2}\frac{\alpha m_{Z^*}^2}{48}\frac{1}{\sin^2\theta_W\cos^2\theta_W}[(2T_{3L} - 4Q\sin^2\theta_W)^2 + 4T_{3L}^2] \\ &\quad \cdot\frac{\alpha p_{cm}}{6\sin^2\theta_W\cos^2\theta_W}\left[\frac{p_{cm}^2}{m^{*2}} + 3\frac{m^2}{m^{*2}}\right] \\ &= \frac{\pi\alpha^2}{192\sin^4\theta_W\cos^4\theta_W}\frac{2p_{cm}\sqrt{s}}{(s-m_Z^2)^2}[(1-4\sin^2\theta_W)^2 + 1][4\frac{p_{cm}^2}{s} + 12\frac{m^2}{s}]\end{aligned}$$

A la recherche du temps perdu...



WW and ZZ fusion in e^+e^-



$$q_1 = p_1 - p_1'$$
$$q_2 = p_2 - p_2'$$

Analogous to two-photon process; $e^+e^- \to e^+e^-X$

Two-photon has transverse photons

WW dominated by longitudinal bosons

Calculation of $e^+e^- \to H\nu\overline{\nu}$

$$-i\mathcal{M} = (igm_W)g_{\mu\nu}\frac{ig}{\sqrt{2}}\frac{\overline{u}(p_1')\gamma^\mu\frac{1}{2}(1-\gamma_5)u(p_1)}{(q_1^2-m_W^2)}\frac{ig}{\sqrt{2}}\frac{\overline{u}(p_2')\gamma^\nu\frac{1}{2}(1-\gamma_5)u(p_2)}{(q_2^2-m_W^2)}$$

$$|\mathcal{M}|^2 = g^6 m_W^2 \frac{1}{4}\frac{1}{4}\frac{\mathrm{Tr}\ [\not{p}_1'\gamma^\mu\not{p}_1\gamma^\nu\frac{1}{2}(1-\gamma_5)]}{(q_1^2-m_W^2)^2}\frac{\mathrm{Tr}\ [\not{p}_2'\gamma^\mu\not{p}_2\gamma^\nu\frac{1}{2}(1-\gamma_5)]}{(q_2^2-m_W^2)^2}$$

Trace stuff

$$|\mathcal{M}|^2 = g^6 m_W^2 \frac{1}{4}\frac{1}{4}\frac{16 p_1'\cdot p_2'\ p_1\cdot p_2}{(q_1^2-m_W^2)^2(q_2^2-m_W^2)^2}$$

write the four-momenta explicitly,

$$p_1 = E(1,0,0,1); \qquad p_2 = E(1,0,0,-1)$$

$$p_1' = (\sqrt{x_1^2E^2+p_{\perp 1}^2}, \vec{p}_{\perp 1}, x_1E); \quad p_2' = (\sqrt{x_2^2E^2+p_{\perp 2}^2}, \vec{p}_{\perp 2}, -x_2E)$$

we find

$$\begin{aligned} q_1^2 &= -2p_1 \cdot p_1' \approx -\frac{p_{\perp 1}^2}{x_1} & q_2^2 &= -2p_2 \cdot p_2' \approx -\frac{p_{\perp 2}^2}{x_2} \\ 2p_1' \cdot p_2' &= x_1 x_2 s; & 2p_1 \cdot p_2 &= s \end{aligned}$$

four-momentum of the Higgs boson, $k = q_1 + q_2$

$$\begin{aligned} m_H^2 &= (q_1+q_2)^2 = \left(2E - \sqrt{x_1^2E^2 + p_{\perp 1}^2} - -\sqrt{x_2^2E^2 + p_{\perp 2}^2}\right)^2 \\ &\quad -(x_1E - x_2E)^2 - (\vec{p}_{\perp 1} + \vec{p}_{\perp 2})^2 \\ &\approx 4E^2(1-x_1)(1-x_2) - (2-x_1-x_2)\left(\frac{p_{\perp 1}^2}{x_1} + \frac{p_{\perp 2}^2}{x_2}\right) - (\vec{p}_{\perp 1} + \vec{p}_{\perp 2})^2 \end{aligned}$$

Do the phase space calculation explicitly, noting that the flux factor, is just $s/2$:

$$
\begin{aligned}
d\sigma &= \frac{(2\pi)^4}{2s}\frac{d^3p_1'}{(2\pi)^3 2p_1'}\frac{d^3p_2'}{(2\pi)^3 2p_2'}\frac{d^3k}{(2\pi)^3 2E_H} \\
&\quad \times\delta^4(p_1'+p_2'+k-p_1-p_2)\frac{s^2x_1x_2}{4} \\
&\quad \times\frac{g^6m_W^2}{(q_1^2-m_W^2)^2(q_2^2-m_W^2)^2}
\end{aligned}
$$

We get rid of the Higgs momentum in the usual way by writing

$$
\frac{d^3k}{(2\pi)^3 2E_H} = \frac{d^4k}{(2\pi)^3}\delta(k^2-m_H^2)
$$

and then integrating d^4k

$$
d\sigma = \frac{g^6m_W^2}{(2\pi)^5 2s}\frac{d^3p_1'}{(2p_1'}\frac{d^3p_2'}{2p_2'}\frac{s^2x_1x_2}{4}\frac{\delta((q_1+q_2)^2-m_H^2)}{\left(\frac{p_{\perp 1}^2}{x_1}+m_W^2\right)^2\left(\frac{p_{\perp 2}^2}{x_2}+m_W^2\right)^2}
$$

Assume $p_\perp$ doesn't influence phase space too much

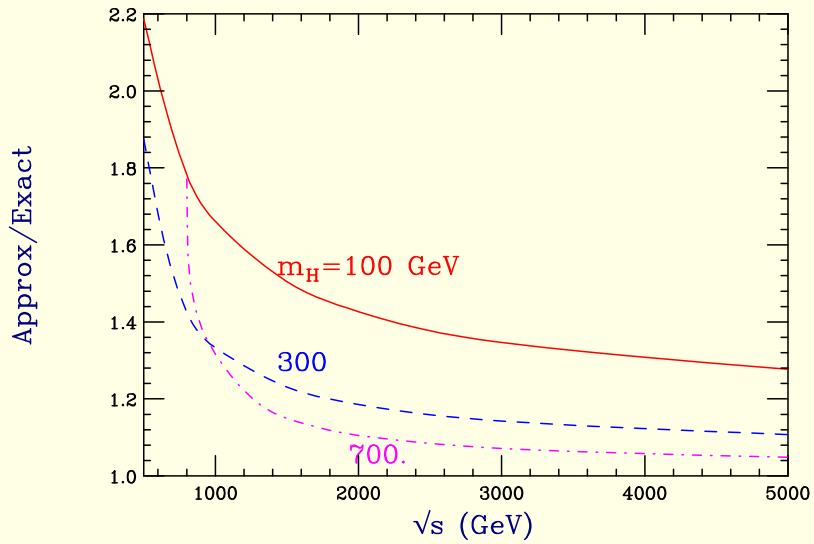
$$\frac{1}{16\pi^2}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 s m_W^2 \frac{dx_1 dx_2 d^2p_{\perp 1} d^2p_{\perp 2}}{\left(\frac{p_{\perp 1}^2}{x_1}+m_W^2\right)^2\left(\frac{p_{\perp 2}^2}{x_2}+m_W^2\right)^2}\delta(s(1-x_1)(1-x_2)-M_H^2)$$

Now integrate to find total cross section

$$\begin{aligned}
d\sigma &= \frac{1}{16}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 s m_W^2 \frac{x_1}{m_W^2}\frac{x_2}{m_W^2} dx_1 dx_2 \delta(s(1-x_1)(1-x_2)-M_H^2) \\
\sigma &= \frac{1}{16}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 \frac{s}{m_W^2}\int_0^{1-m_H^2/s} dx \frac{x}{s(1-x)}\left(1-\frac{m_H^2}{s(1-x)}\right) \\
&= \frac{1}{16 m_W^2}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 \left[\left(1+\frac{m_H^2}{s}\right)\ln\frac{s}{m_H^2} - 2\left(1-\frac{m_H^2}{s}\right)\right]
\end{aligned}$$

Features of W Fusion

- Particle with mass m_H is being produced, but cross section not suppressed by a factor $1/m_H^2$.
- The cross section is "anomalously large."
- Recoil quarks have $p_\perp \sim \mathcal{O}(m_W)$: use for tagging
- Approximation overestimates cross section



Higgs Production at Hadron Colliders

Hadron beam is really a beam of quarks, anitquarks, and gluons

Cross sections obtained by convoluting with parton distributions

$$d\sigma = \int dx_1 dx_2 f_1(x_1) f_2(x_2) d\hat{\sigma}$$

For resonance production

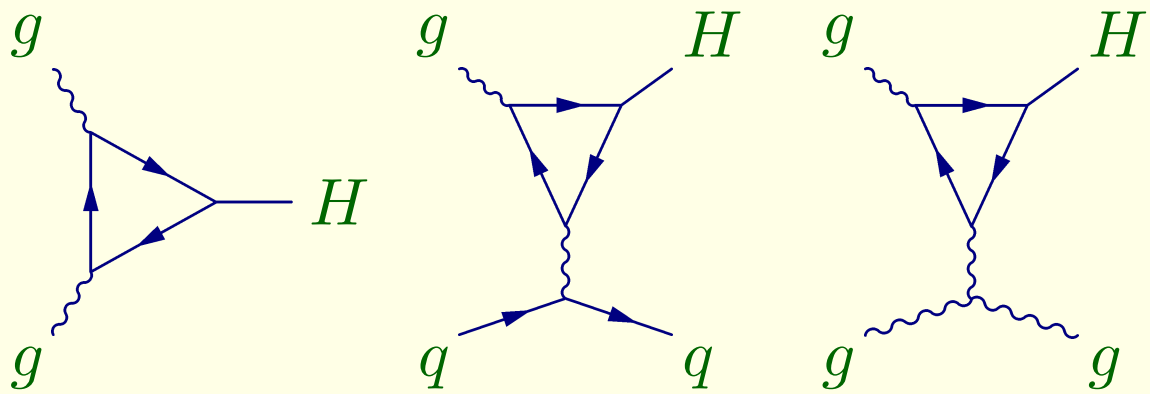
$$\begin{aligned} \hat{\sigma} &= \frac{2J+1}{(2S_a+1)(2S_b+1)} \frac{4\pi}{k^2} \frac{\Gamma^2/4}{(E-M)^2+\Gamma^2/4} BR(R \to ab) \\ &\to (2J+1)\frac{4\pi^2}{m}\Gamma(R \to ab)\delta(\hat{s}-m^2) \end{aligned}$$

Note: always two polarizations for quarks, gluons

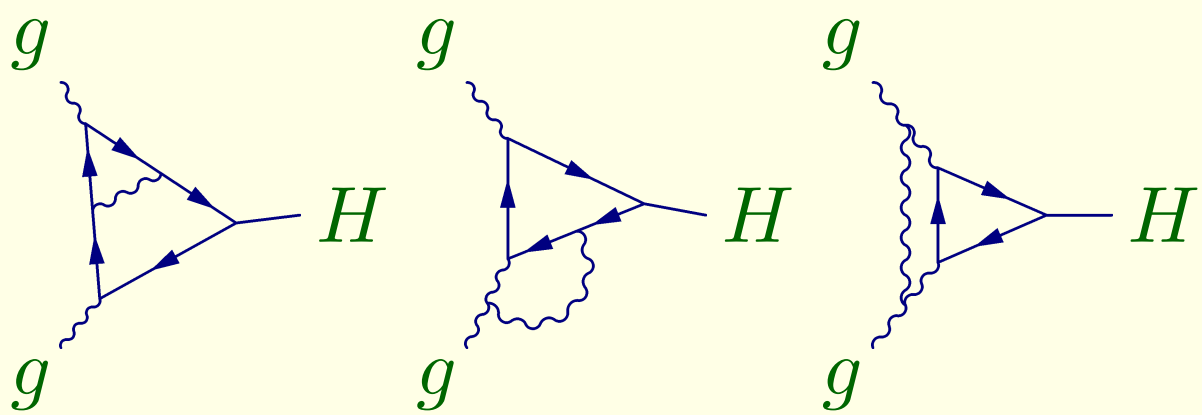
$$\sigma = \frac{(2J+1)4\pi^2}{m^3}\Gamma(R \to ab)\tau\frac{d\mathcal{L}}{d\tau} \qquad [\tau = m^2/s]$$

Higgs Production at Hadron Colliders

Dominant cross section is from gluon fusion



Lowest order $gg \to H$, $qg \to qH$, and $gg \to Hg$



Some virtual radiative corrections to $gg \to H$.

Gluon Fusion

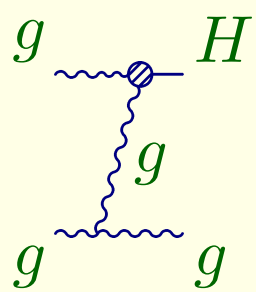
$$\sigma = \left(\frac{1}{8} \cdot \frac{1}{8} \cdot 2\right) \frac{4\pi^2}{m_H} \Gamma(H \to gg) \delta(\hat{s} - m_H^2)$$

- Factors of 8: "average over initial, sum over final"
- Factor of 2: "divide by 2 for identical particles in final state"
- Now approximate: t contribution only, $m_t >> m_H$ (actually ok)

$$\sigma = \frac{\alpha_s^2}{576\pi v^2} \tau \frac{d\mathcal{L}}{d\tau}$$

Radiative corrections

- Contributions from $qg \to Hq$, $q\bar{q} \to Hg$ small
- Combine $gg \to H$, $gg \to Hg$ with evolution of structure functions



- Partition bremsstrahlung: large $p_\perp$ into scattering, small $p_\perp$ into beam evolution
- Combination ultimately independent of renormalization scale

Simple treatment of radiative corrections

Include only t. Take $m_t \to \infty$

$$\hat{\sigma}(gg \to hX) = \frac{\alpha_s^2}{576\pi v^2}\left\{\delta(1-z) + \frac{\alpha_s(\mu)}{\pi}\left[h(z) + \overline{h}(z)\log\left(\frac{m_h^2}{\mu^2}\right)\right]\right\}$$

where $z = m_H^2/\hat{s}$ and

$$h(z) = \delta(1-z)\left(\pi^2 + \frac{11}{2}\right) + \text{messy stuff} \quad \overline{h}(z) = \text{other messy stuff}$$

Introduce K factor : $K = \frac{\sigma(pp \to HX)}{\sigma(pp \to H)}$

K between two and three at LHC - arises largely from a rescaling

$$\begin{aligned} K_{rescaling} &= 1 + \frac{\alpha_s(\mu)}{\pi}\left(\pi^2 + \frac{11}{2}\right) \\ &= 1 + 4.89\,\alpha_s(\mu) \end{aligned}$$

W and Z content of fermions

Think of Ws as partons in fermion

$$\frac{1}{16\pi^2}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 s m_W^2 \frac{dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}}{\left(\frac{p_{\perp 1}^2}{1-y_1}+m_W^2\right)^2\left(\frac{p_{\perp 2}^2}{1-y_2}+m_W^2\right)^2}$$

$$= \frac{4\pi^2}{m}\Gamma(R\to ab) f_a(y_1) f_b(y_2)$$

y is fraction of fermion's momentum given to W

Use $\Gamma(H\to W_L W_L) = \frac{G_F}{\sqrt{2}}\frac{m_H^3}{8\pi} = \frac{g^2}{8m_W^2}\frac{m_H^3}{8\pi} = \frac{4\pi\alpha}{8\sin^2\theta_W m_W^2}\frac{m_H^3}{8\pi}$

to find the distribution of the W partons inside the electron:

$$\frac{g^2}{16\pi^3}\frac{dy}{y}\frac{d^2p_\perp}{[m_W^2+p_\perp^2/(1-y)]^2} \longrightarrow \frac{g^2}{16\pi^2}(1-y)\frac{dy}{y}$$

Use this to study WW, WZ, etc. scattering

WW Scattering

If Higgs turns out to be very massive, λ is large

This means strong interactions between longitudinal Ws Zs

Equivalence Theorem: replace longitudinal W, Z with scalars

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where

$$\phi^\dagger \phi = \tfrac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

Then if $\mu^2 < 0$, some expectation value is non-zero. We write

$$\begin{aligned} \phi_3 &= \sigma = \langle \phi_3 \rangle + H = \langle \sigma \rangle + H \\ v^2 &= -\mu^2/\lambda \end{aligned}$$

Rewrite the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu H\partial^\mu H + \mu^2 H^2 \\
&\qquad + \frac{\mu^2}{v} H(H^2+\pi^2) - \lambda(H^2+\pi^2)^2 \\
&= \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu H\partial^\mu H - m_H^2 H^2 \\
&\qquad - \frac{m_H^2}{2v} H(H^2+\pi^2) - \frac{m_H^2}{8v^2}(H^2+\pi^2)^2
\end{aligned}
$$

Identify

$$\pi^2 = 2w^+w^- + zz$$

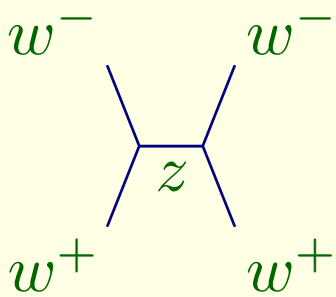
Example of Equivalence Thm: $H \to WW$

$$-i\mathcal{M} = -i\frac{m_H^2}{2v} \cdot 2$$

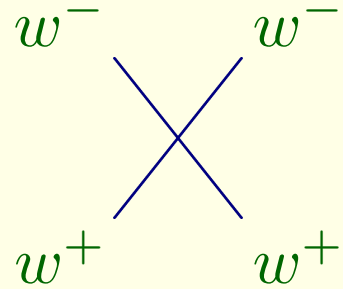
$$\begin{aligned}
\Gamma &= \frac{1}{8\pi}\frac{p}{M^2}|\mathcal{M}|^2 \\
&\approx \frac{1}{8\pi}\frac{m_H/2}{m_H^2}\frac{m_H^4}{v^2} \\
&= \frac{1}{8\pi}\frac{G_F}{\sqrt{2}}m_H^3
\end{aligned}$$

Agrees with our result for $m_H \to \infty$

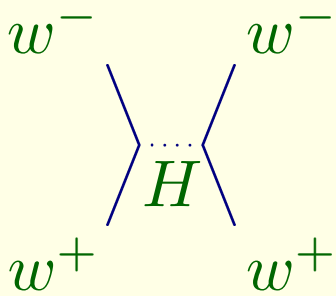
$w^+w^- \to w^+w^-$



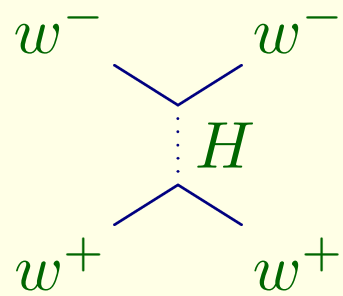
$$-i\mathcal{M} = 0$$



$$-i\mathcal{M} = -\frac{im_H^2}{8v^2} \cdot 4 \cdot 2 \cdot 2 = -\frac{2im_h^2}{v^2}$$



$$-i\mathcal{M} = \left(-\frac{im_H^2}{2v}\right)^2 \frac{i}{s-m_H^2} \cdot 2 \cdot 2$$

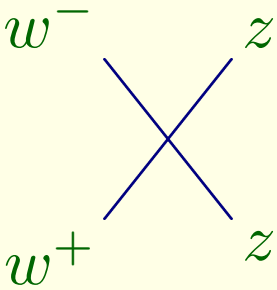


$$-i\mathcal{M} = \left(-\frac{im_H^2}{2v}\right)^2 \frac{i}{t-m_H^2} \cdot 2 \cdot 2$$

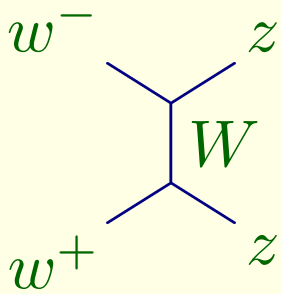
For the sum, we find

$$-i\mathcal{M} = -\frac{im_H^2}{v^2}\left(2 + \frac{m_H^2}{s-m_H^2} + \frac{m_H^2}{t-m_H^2}\right) \approx -\frac{iu}{v^2}$$

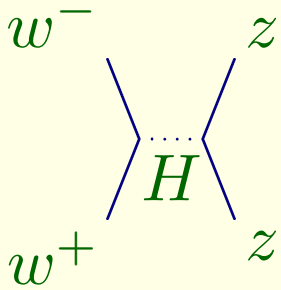
$w^+w^- \to zz$



$$-i\mathcal{M} = -\frac{im_H^2}{8v^2} \cdot 4 \cdot 2 = -\frac{im_H^2}{v^2}$$



$$-i\mathcal{M} = 0$$

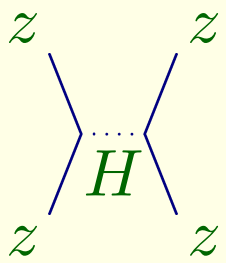


$$-i\mathcal{M} = \frac{i}{s-m_H^2}\left(-\frac{im_H^2}{2v}\right)^2 \cdot 2 \cdot 2 = -\frac{im_H^4}{v^2(s-m_H^2)}$$

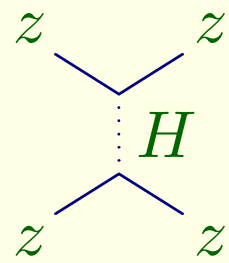
Summing these we find

$$-i\mathcal{M} = -\frac{im_H^2}{v^2}\left(1 + \frac{m_H^2}{s-m_H^2}\right) = -\frac{im_H^2}{v^2}\frac{s}{s-m_H^2} \approx \frac{is}{v^2}$$

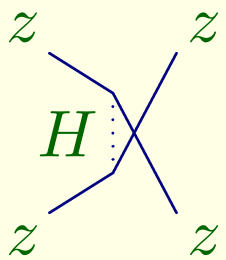
$zz \longrightarrow zz$



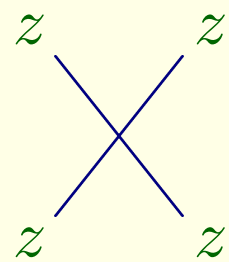
$$-i\mathcal{M} = \frac{i}{s-m_H^2}\left(-\frac{im_H^2}{2v}\right)^2 \cdot 2 \cdot 2$$



$$-i\mathcal{M} = \frac{i}{t-m_H^2}\left(-\frac{im_H^2}{2v}\right)^2 \cdot 2 \cdot 2$$



$$-i\mathcal{M} = \frac{i}{u-m_H^2}\left(-\frac{im_H^2}{2v}\right)^2 \cdot 2 \cdot 2$$



$$-i\mathcal{M} = -\frac{im_H^2}{8v^2} \cdot 4!$$

Here the sum is

$$-i\mathcal{M} = -\frac{m_H^4}{v^2}\left(\frac{i}{s-m_H^2} + \frac{i}{t-m_H^2} + \frac{i}{u-m_H^2}\right) - 3i\frac{m_H^2}{v^2} = 0$$

s-wave amplitudes

Born amplitudes have s-wave and p-wave

$$\begin{aligned}
\mathcal{M}(w^+w^- \to w^+w^-) &= \frac{u}{v^2} \\
\mathcal{M}(w^+w^+ \to w^+w^+) &= \frac{s}{v^2} \\
\mathcal{M}(w^+w^- \to zz) &= -\frac{s}{v^2} \\
\mathcal{M}(zz \to zz) &= 0
\end{aligned}$$

since $t = -s(1-\cos\theta)/2; u = -s(1+\cos\theta)/2$
with

$$\mathcal{A} = -\frac{1}{16\pi}\frac{2k}{\sqrt{s}}\mathcal{M} = -\frac{1}{16\pi}\mathcal{M}$$

the s-wave amplitudes are

$$
\begin{aligned}
\mathcal{A}(w^+w^+ \to w^+w^+) &= -s/s_0 \\
\mathcal{A}(w^+w^- \to w^+w^-) &= \frac{1}{2}s/s_0 \\
\mathcal{A}(w^+w^- \to zz) &= s/s_0 \\
\mathcal{A}(z \to zz) &= 0
\end{aligned}
$$

where $s_0 = 16\pi v^2 = (1.7\ \mathrm{TeV})^2$

s-wave Unitarity

Normally

$$\Im\mathcal{A} = |\mathcal{A}|^2; \qquad \Im(1/\mathcal{A}) = -1$$

so that

$$\mathcal{A} = e^{i\delta}\sin\delta$$

For identical boson scattering

$$\Im\mathcal{A} = \frac{1}{2}|\mathcal{A}|^2; \qquad \Im(1/(\mathcal{A}/2)) = -1$$

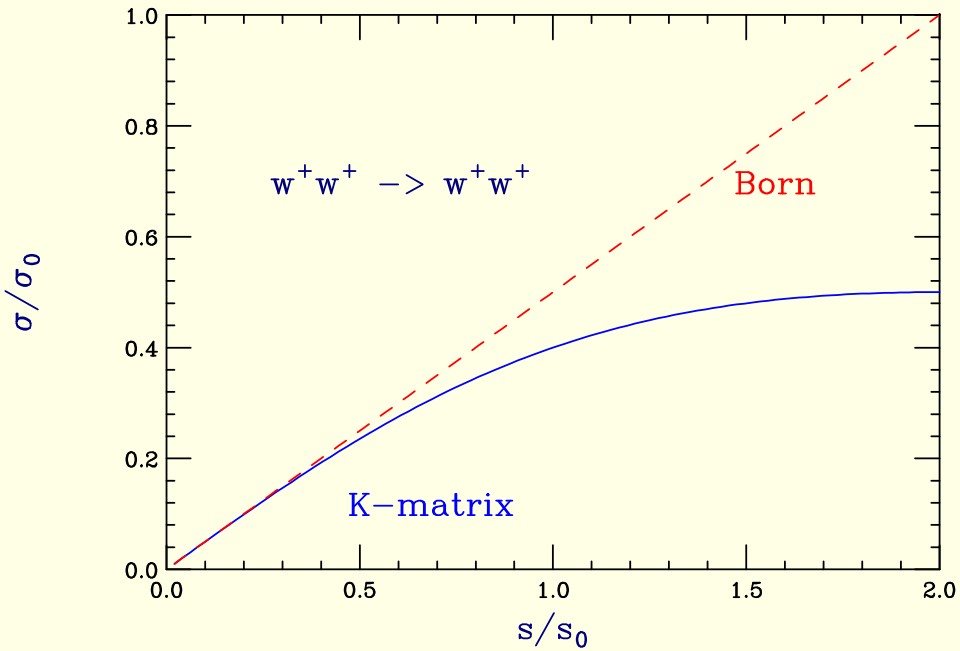
so instead

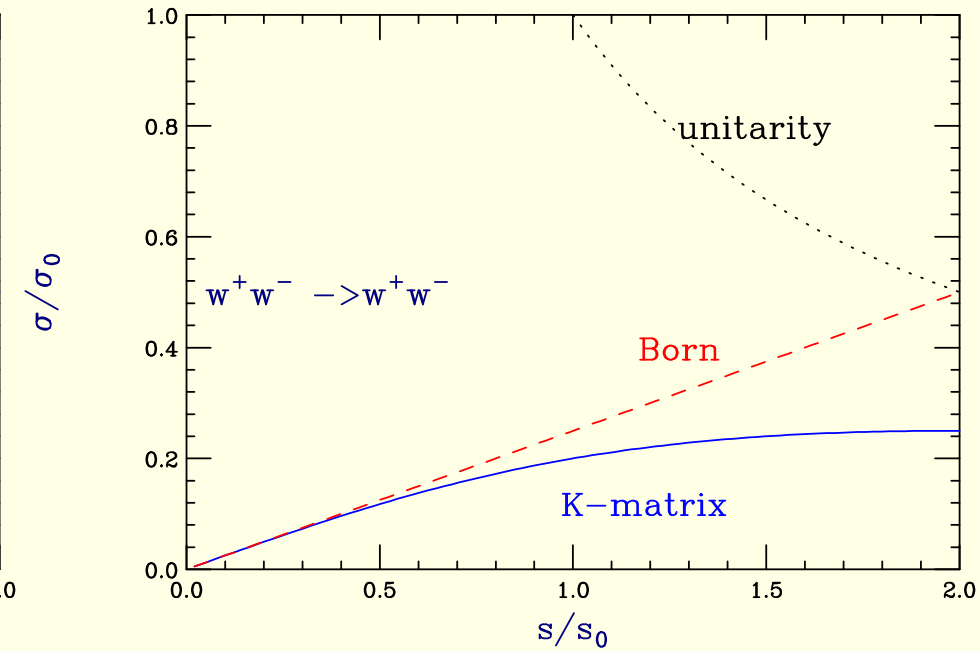
$$\mathcal{A} = 2e^{i\delta}\sin\delta$$

In the ordinary case make a Born amplitude unitary by K matrix

$$(1/a) = (1/a_{Born}) - i$$

Adapt here, including isospin.


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
σ/σ0
s/s0
w+w+ -> w+w+
Born
K−matrix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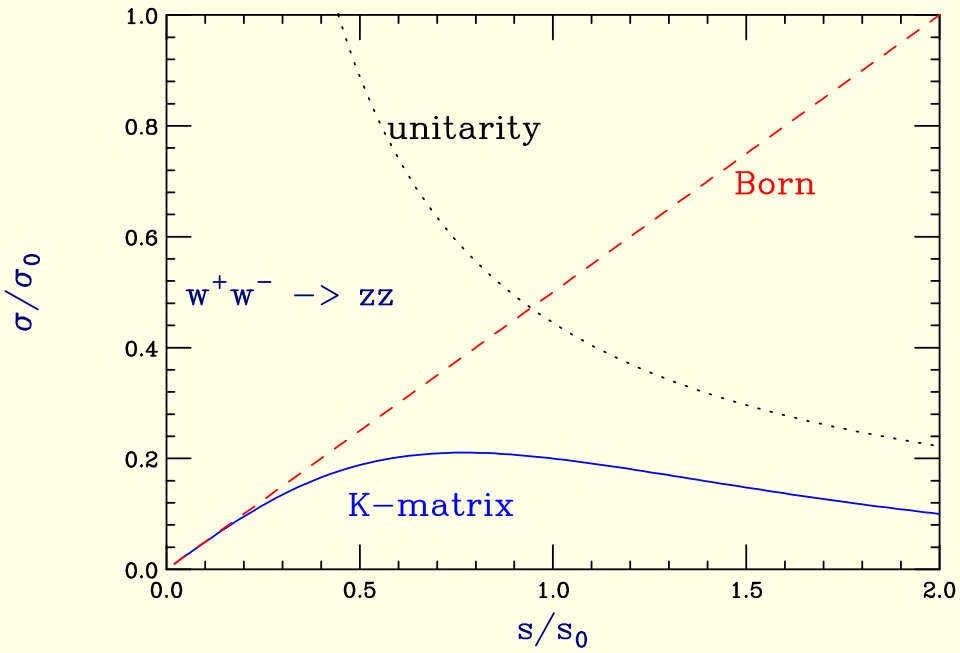
2.0
σ/σ0
s/s0
unitarity
w+w− ->w+w−
Born
K−matrix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s/s0
unitarity
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w+w− −> zz
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