Higgs Boson - Lecture 1

Spontaneous Symmetry Breaking and Gauge Theories

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Le Higgs: La Chasse Continue!

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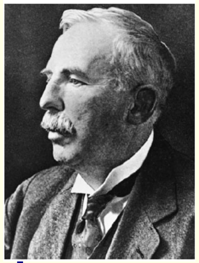
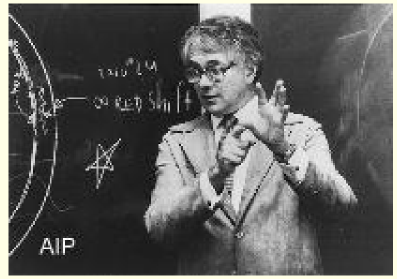
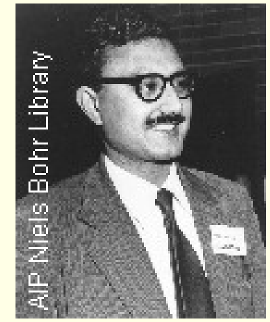
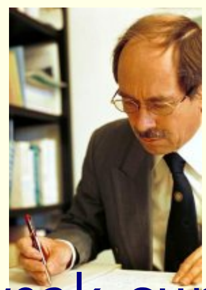
History of Electroweak Interactions

- 1898: Rutherford 
- 1930: Pauli neutrino 
- 1934: Fermi theory 
- 1954: Yang-Mills
- 1956: Lee-Yang: Parity violation?  
- 1956: C. S. Wu: Parity violation! 

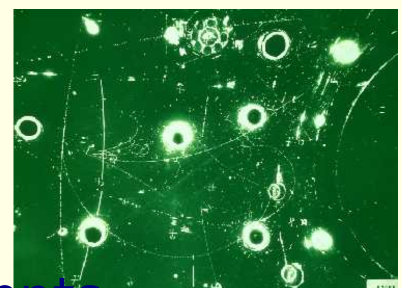





- 1957: $V - A$
- 1961: Glashow; 1967: Weinberg; 1968: Salam
- 1964: Higgs
- 1971: ‘t Hooft



- 1973: Gargamelle: neutral weak currents



- 1978: SLAC: parity violating neutral weak currents
- 1983: CERN: W and Z

Things are Not What They Seem

A Simple Scalar Field -

$$\mathcal{L} = \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{2}\lambda\phi^4$$

Symmetry: $\phi \rightarrow -\phi$ - no cubic coupling

What if $\mu^2 < 0$?

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\lambda\phi^4$$

has a minimum when

$$\phi^2 = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

Expand About Minimum

$$\phi = \frac{v}{\sqrt{2}} + \rho$$

In terms of the new field,

$$\mathcal{L} = \frac{1}{2}\partial_\alpha\rho\partial^\alpha\rho - \frac{3}{8}\frac{\mu^4}{\lambda} + \mu^2\rho^2 - \sqrt{\frac{-\mu^2\lambda}{2}}\rho^3 - \frac{\lambda}{2}\rho^4$$

New scalar with $m^2 = -2\mu^2$ and cubic couplings. Parity broken.

You can't tell the particle content by reading the labels!

Even better, try four scalar fields: $\phi_i, i = 0, \ldots 3$

$$\mathcal{L} = \partial_\alpha \phi_i \partial^\alpha \phi_i - \frac{1}{2}\mu^2 \phi_i \phi_i - \frac{1}{2}\lambda(\phi_i \phi_i)^2$$

It is clear that the minimum of the potential energy occurs for

$$\phi_i \phi_i = -\frac{\mu^2}{2\lambda}$$

Write $\sigma = \phi_0$, $\pi_i = \phi_i; i = 1, 2, 3$

$$V = \frac{\mu^2}{2}(\sigma^2 + \pi_i \pi_i) + \frac{\lambda}{2}(\sigma^2 + \pi_i \pi_i)^2$$

Take vacuum expectation along $\sigma: \quad \sigma = v + \rho$

Goldstone Bosons

Expect one massive boson with $m^2 = -2\mu^2$, but what else?

$$-\frac{\mu^2}{2}(\pi_i\pi_i) - \frac{\lambda}{2}(\langle\sigma\rangle^2 + \pi_i\pi_i)^2 = -\frac{\mu^2}{2}(\pi_i\pi_i) - \frac{\lambda}{2}(-\frac{\mu^2}{2\lambda} + \pi_i\pi_i)^2$$

All the $\pi_i\pi_i$ terms vanish! The pions are massless.

Counting Goldstone Bosons

Goldstone's theorem: For every symmetry generator broken spontaneously there is a massless scalar boson.

We had $O(4)$ symmetry with $4 \times 3/2 = 6$ generators.

Only $O(3)$ with $3 \times 2/2 = 3$ generators survived.

There must be $6 - 3 = 3$ Goldstone bosons.

Another view

Complex doublet, like the kaons

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_2 \\ \phi_3 - i\phi_4 \end{pmatrix}; \qquad \overline{\phi} = \begin{pmatrix} \overline{\phi}^0 \\ -\phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 + i\phi_4 \\ -(\phi_1 + i\phi_2) \end{pmatrix}$$

Conjugates:

$$\begin{aligned} \phi^\dagger &= \left(\phi^-,\ \overline{\phi}^0\right) = \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2,\ \phi_3 + i\phi_4)\,; \\ \overline{\phi}^\dagger &= \left(\phi^0,\ -\phi^+\right) = \frac{1}{\sqrt{2}} (\phi_3 - i\phi_4,\ -\phi_1 + i\phi_2) \end{aligned}$$

Invariant product:

$$\phi^\dagger \phi = \overline{\phi}^\dagger \overline{\phi} = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

What You See Is Not What You Get

Lagrangian of charged scalar (e.g. $\pi^{\pm}$): $\phi = (\phi_1 - i\phi_2)/\sqrt{2}$

$$\mathcal{L} = \partial_\alpha \phi^\dagger \partial^\alpha \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

Lagrangian for electromagnetism is

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Combine them with “minimal coupling”

$$\partial_\alpha \to \partial_\alpha + ieA_\alpha$$

so that we have

$$\begin{aligned}\mathcal{L} &= (\partial_\alpha - ieA_\alpha)\phi^\dagger(\partial^\alpha + ieA^\alpha)\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\ &\quad -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\end{aligned}$$

Higgs Mechanism

Gauge invariance:

$$\phi \to e^{-i\theta(x)}\phi; \qquad A_\alpha \to A_\alpha + \tfrac{1}{e}\partial_\alpha\theta$$

leaves Lagrangian unchanged.

Now let $\mu^2 < 0$ so ϕ gets vacuum expectation

$$\langle |\phi|^2 \rangle = \tfrac{-\mu^2}{2\lambda} \equiv \tfrac{v^2}{2}$$

$$\phi = \tfrac{v+\rho}{\sqrt{2}} e^{i\chi(x)/v}$$

we can rotate $\chi(x)$ away with gauge transformation

$$\phi = \tfrac{v+\rho}{\sqrt{2}}$$

Goldstone theorem predicts massless scalar because $\langle\phi\rangle$ breaks the rotation ($U(1)$) symmetry

Goldstone's Theorem is Wrong

Lagrangian in terms of new field, ρ

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}|(\partial_\alpha + ieA_\alpha)(v+\rho)|^2 - \frac{1}{2}\mu^2(v+\rho)^2 \\
&\quad -\frac{1}{4}\lambda(v+\rho)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&= \frac{1}{2}\partial_\alpha\rho\partial^\alpha\rho - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2\frac{\mu^2}{2}\rho^2 + \frac{1}{2}e^2v^2A_\mu A^\mu \ldots
\end{aligned}$$

Find a scalar with $m_H^2 = -2\mu^2$ but no massless particle.

Instead the photon gets massive: $m_V^2 = e^2v^2$.

The photon ate the Goldstone boson and became massive.

This is the Higgs mechanism (P. Higgs, 1964).

Gauge covariant derivative

Fields are "rotated" by gauge transformation

$$\phi \rightarrow e^{-i\chi Q}\phi$$

Gauge invariance achieved with "covariant derivative"

$$\begin{aligned} D_\mu\phi &= (\partial_\mu + ieQA_\mu)\phi \rightarrow (\partial_\mu + ieQA_\mu + ie\frac{1}{e}Q\partial_\mu\chi)e^{-i\chi Q}\phi \\ &= e^{-i\chi Q}D_\mu\phi \end{aligned}$$

Easy to make gauge invariant combinations in Lagrangian

Yang-Mills

Yang-Mills theory: generators T^a

$$[T^a, T^b] = i f_{abc} T^c$$

For example, rotation group has $f_{ijk} = \epsilon_{ijk}$

A multiplet, ϕ_i, is "rotated"

$$\phi \to e^{-i\chi^a T^a} \phi$$

Can we find a covariant derivative here, too? Yes

$$D_\mu \phi = (\partial_\mu + igT \cdot A_\mu)\phi$$

Together with the rule

$$A_\mu \to A_\mu + \delta A_\mu; \quad T \cdot \delta A_\mu = \frac{i}{g}(\partial_\mu e^{-i\chi \cdot T}) e^{i\chi \cdot T}$$

gives just what we want:

$$D_\mu \phi \to e^{-i\chi^a T^a} D_\mu \phi$$

Homework: prove this for yourself!

Field Strength

We need to generalize $F_{\mu\nu}$, too. The answer is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

To see this is right, notice

$$D_\mu \to e^{-i\chi\cdot T} D_\mu \; e^{i\chi\cdot T}$$

so

$$[D_\mu, D_\nu] \to e^{-i\chi\cdot T}[D_\mu, D_\nu] e^{i\chi\cdot T}$$

This means we'll get the right properties if we take

$$[D_\mu, D_\nu] = ig[\partial_\mu T\cdot A_\nu - \partial_\nu T\cdot A_\mu + ig[T\cdot A_\mu, T\cdot A_\nu] \equiv igT\cdot F_{\mu\nu}$$

Yang-Mills + Higgs

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

Take $\mu^2 < 0.$ At the minimum

$$\langle\phi\rangle^\dagger\langle\phi\rangle = \frac{-\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

Consider effect of just vacuum

$$D_\mu\langle\phi\rangle = igA^a_\mu T^a\langle\phi\rangle$$

Makes quadratic term - mass matrix

$$\frac{1}{2}g^2A^a_\mu A^{b\mu}(T^a\langle\phi\rangle)^\dagger(T^b\langle\phi\rangle) = \frac{1}{2}m^2_{ab}A^a_\mu A^{b\mu}$$

Symmetry Breaking Patterns

If $T^0\langle\phi\rangle = 0$, then the whole column and row vanish.
Corresponding vector remains massless.

- Minimize potential energy, picking appropriate $\langle\phi\rangle$
- The T's such that $T\langle\phi\rangle = 0$ give surviving symmetry
- Each such generator gives massless gauge boson
- Other gauge bosons become massive by eating Goldstone bosons

The Standard Model

- In 1960s, needed just three gauge bosons, $W^{\pm}$ and A
- Could try $SU(2) = O(3)$: three generators. This doesn't work [why?]
- Actually use $SU(2) \times U(1)$
- Generators for $SU(2)$ are T^{+}, T^{-}, T_3. Generator for U(1): Y
- Some combination must be electric charge $Q = T_3 + Y/2$
- Borrows notation from Gell-Mann - Nishijima relation

The Real Thing

Covariant derivative

$$D_\mu = \partial_\mu + ig\vec{T}\cdot\vec{W}_\mu + ig'\tfrac{Y}{2}B_\mu$$

or

$$D_\mu = \partial_\mu + i\tfrac{g}{\sqrt{2}}[T^+W_\mu^+ + T^-W_\mu^-] + igT_3W_{3\mu} + ig'\tfrac{Y}{2}B_\mu$$

add complex scalar (weak)isodoublet

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

Spontaneous Breakdown

Use $SU(2)$ to rotate ϕ to "down"

Use $U(1)$ to make this component real

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\rho(x) \end{pmatrix}$$

with

$$v^2 = \frac{-\mu^2}{\lambda}$$

$\rho(x)$ is the Higgs boson

Gauge Boson Mass Matrix

To find the mass squared matrix of the vector bosons we compute

$$
\begin{aligned}
D_\mu\langle\phi\rangle &= \left\{ i\frac{g}{\sqrt{2}}[T^+W_\mu^+ + T^-W_\mu^-] + igT_3W_{3\mu} + ig'\frac{Y}{2}B_\mu \right\} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} igW^+v/2 \\ -i\frac{gv}{2\sqrt{2}}W_3 + i\frac{g'v}{2\sqrt{2}}B \end{pmatrix}
\end{aligned}
$$

It is easy now to find

$$(D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle) = \frac{g^2v^2}{4}W^+W^- + \frac{(g^2+g'^2)v^2}{8}\left(\frac{gW_3-g'B}{\sqrt{g^2+g'^2}}\right)^2$$

These provide the masses for $W^\pm$ and Z:

$$M_W^2 = \frac{g^2v^2}{4}; \qquad M_Z^2 = \frac{(g^2+g'^2)v^2}{4}$$

Z and the Mixing Angle

W_3 and B mix. Massive state is

$$Z = \cos\theta_W W_3 - \sin\theta_W B$$

while the orthogonal combination

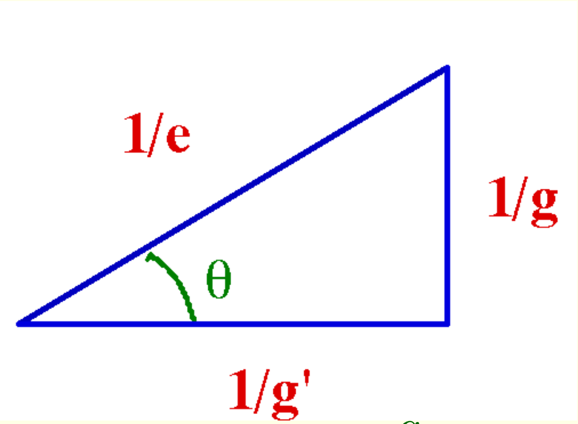
$$A = \cos\theta_W B + \sin\theta_W W_3$$

is the massless photon.
Define $\sin\theta_W$ by

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\begin{aligned} W_3 &= \sin\theta_W A + \cos\theta_W Z \\ B &= \cos\theta_W A - \sin\theta_W Z \end{aligned}$$

Coupling of the Z



$$
\begin{aligned}
D_\mu &= \partial_\mu + i\frac{g}{\sqrt{2}}[T^+W_\mu^+ + T^-W_\mu^-] + igT_3[\sin\theta_W A + \cos\theta_W Z] \\
&\qquad + ig'\frac{Y}{2}[\cos\theta_W A - \sin\theta_W Z] \\
&= \partial_\mu + i\frac{g}{\sqrt{2}}[T^+W_\mu^+ + T^-W_\mu^-] + \frac{ig}{\cos\theta_W}(T_3 - Q\sin^2\theta_W)Z_\mu + ig\sin\theta_W Q A_\mu
\end{aligned}
$$

from which we deduce

$$e = g\sin\theta_W; \qquad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

Fermions in the Standard Model

The Dirac equation

$$(i\not\partial - m)\psi = 0 \quad \rightarrow \quad (i\not{D} - m)\psi = 0$$

in the Lagrangian

$$\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi$$

$V - A$ means that left-handed fermions interact with $W^{\pm}$.

Put left-handed fermions in doublets of $SU(2)$

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

Assigment of Fermions

Actually use chirality states

$$e_L = \frac{1}{2}(1-\gamma_5)e; \qquad e_R = \frac{1}{2}(1+\gamma_5)e$$

These are left-handed and right-handed particles only when ultrarelativistic.

	e_L	e_R	ν_L	u_L	u_R	d_L	d_R
T_3	$-1/2$	0	$1/2$	$1/2$	0	$-1/2$	0
$Y/2$	$-1/2$	-1	$-1/2$	$1/6$	$2/3$	$1/6$	$-1/3$
Q	-1	-1	0	$2/3$	$2/3$	$-1/3$	$-1/3$

Usual mass term not permitted

$$\overline{e}e = \overline{e}_L e_R + \overline{e}_R e_L$$

Not $SU(2)$ invariant: combines singlet and doublet

Fermions Masses from Higgs Coupling

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \qquad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \qquad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}; \qquad H^c = \begin{pmatrix} \overline{\phi}^0 \\ -\phi^- \end{pmatrix}$$

$$\overline{L} = (\overline{\nu}_L, \overline{e}_L)\,; \overline{Q} = (\overline{u}_L, \overline{d}_L)\,; H^\dagger = \left(\phi^-, \overline{\phi}^0\right); H^{c\dagger} = \left(\phi^0, -\phi^+\right)$$

and compose $\mathcal{L}_{Yukawa}$ as

$$g_e[\overline{L}He_R + \overline{e}_R H^\dagger L] + g_d[\overline{Q}Hd_R + \overline{d}_R H^\dagger Q] + g_u[\overline{Q}H^c u_R + \overline{u}_R H^{c\dagger} Q]$$

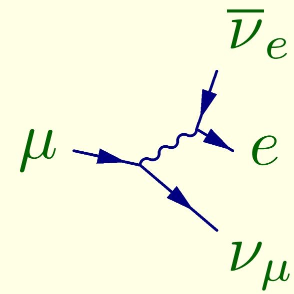
Using just $\langle\phi\rangle$

$$\mathcal{L}_{fermion\ masses} = \frac{v}{\sqrt{2}}[g_e\overline{e}e + g_d\overline{d}d + g_u\overline{u}u]$$

Yukawa couplings connect to the fermion masses

$$m_f = -\frac{g_f v}{\sqrt{2}}$$

The Value of v



Find v by looking at μ decay

$$\left(\frac{g}{\sqrt{2}}\right)^2 \overline{\mu}\frac{\gamma_\lambda}{2}(1-\gamma_5)\nu_\mu \ \frac{1}{M_W^2} \ \overline{\nu}_e\frac{\gamma^\lambda}{2}(1-\gamma_5)e$$

Comparing with the traditional weak interaction Lagrangian

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\overline{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu \ \overline{\nu}_e\gamma^\lambda(1-\gamma_5)e$$

we conclude that

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}; \quad v = 246 \text{ GeV}; \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}$$